MHD is beautiful

OK, if Berlusconi looses tonight

I am willing to give him a 25 h class

in serious MHD

on the other hand...

I have been challenged to show you
that MHD can be more fun than some other stuff

MHD is beautiful

MHD for observers / observations for MHD...

Michel Tagger

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Abstract: forget hand-waving arguments, field-waving is more fun !

my goals:

- (nearly) no equations
- (mostly) ugly cartoons
- understand what MHD does
- understand what MHD can't do
- get a little but broad taste of the physics
- know about numerical simulations

but above all... try to avoid headaches !

Euler equation

- motion of a point mass: $m\frac{d\vec{V}}{dt} = \vec{F}$
- for a fluid: $\rho\frac{d\vec{V}}{dt} = \vec{F}$

(ρ = mass density, g cm^{-3}
F = volumic force)

$$= -\vec{\nabla}p + \rho\,\vec{g} + \frac{1}{c}\,\vec{j} \times \vec{B}$$

pressure gradient

gravity

Lorentz force

Magneto-Hydrodynamics: ions + electrons treated as a single fluid
submitted to usual forces + Lorentz

what happens in reality: 2 fluids, ions and electrons
electrons are much lighter -> carry the current
-> feel Lorentz force
-> transfer it to the ions (which have most of the inertia) by collisions

Closing the system

ρ is given from $\vec{V}$ by the continuity equation (conservation of mass):

$$\frac{d\rho}{dt} = -\rho \, \vec{\nabla}.\vec{V}$$

then p by the equation of state, e.g. polytropic: $p\rho^{-\gamma} = Const.$

and $\vec{B}$ is given by Ohm's law:

$$\underbrace{\vec{E} + \frac{1}{c}\,\vec{V} \times \vec{B}}_{\text{electric field, transformed to the fluid frame}} = \underbrace{\eta}_{\text{resistivity}} \, \vec{j}$$

electric field, transformed to the fluid frame

resistivity

= electron/ion friction

not over yet ! Need to get $\vec{E}$ and $\vec{j}$ from $\vec{B}$, by Maxwell's equations

Maxwell's equations

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j}$$

~~(+ displacement current if relativistic)~~

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

induction equation

Ideal MHD

just as collisions, viscosity, heat transport etc...

resistivity (= magnetic diffusivity) computed from first principles (e-i collisions) is extremely weak -> can often be neglected

$$\longrightarrow \quad \vec{E} + \frac{1}{c}\vec{V} \times \vec{B} = 0$$

coupled with induction:

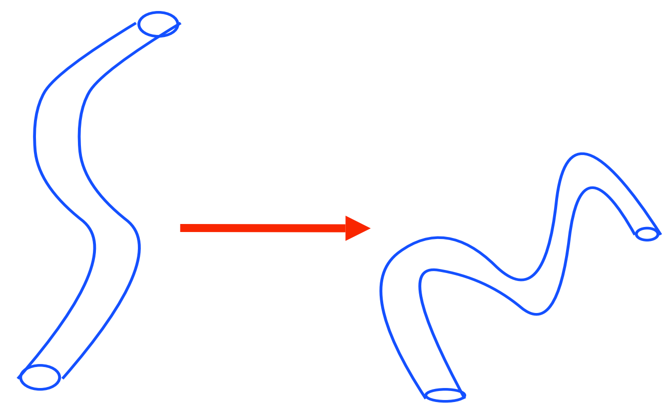
$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

gives:

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times (\vec{V} \times \vec{B}) = 0$$

no demonstration here but this implies:

frozen flux (gas and field lines move together)



conservation of magnetic topology

Already two examples of the role of "frozen flux"

1°) Parker instability

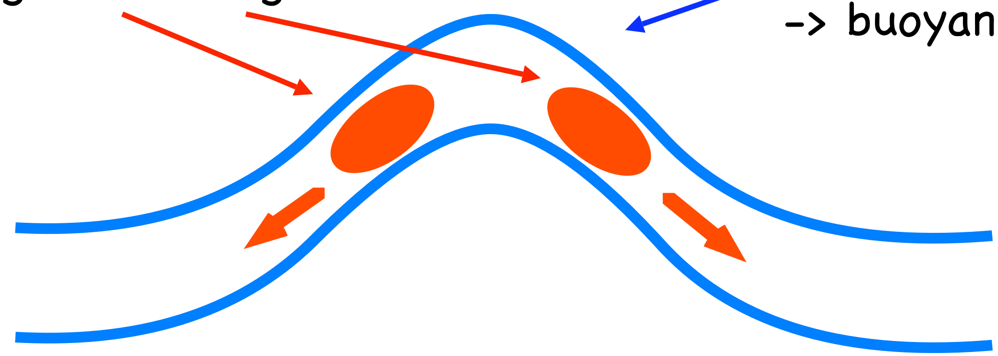
in a stratified, magnetized medium

if a flux tube starts to buckle

gas starts sliding down along B

the top becomes lighter

-> buoyancy pushes it further up



the bottom becomes heavier

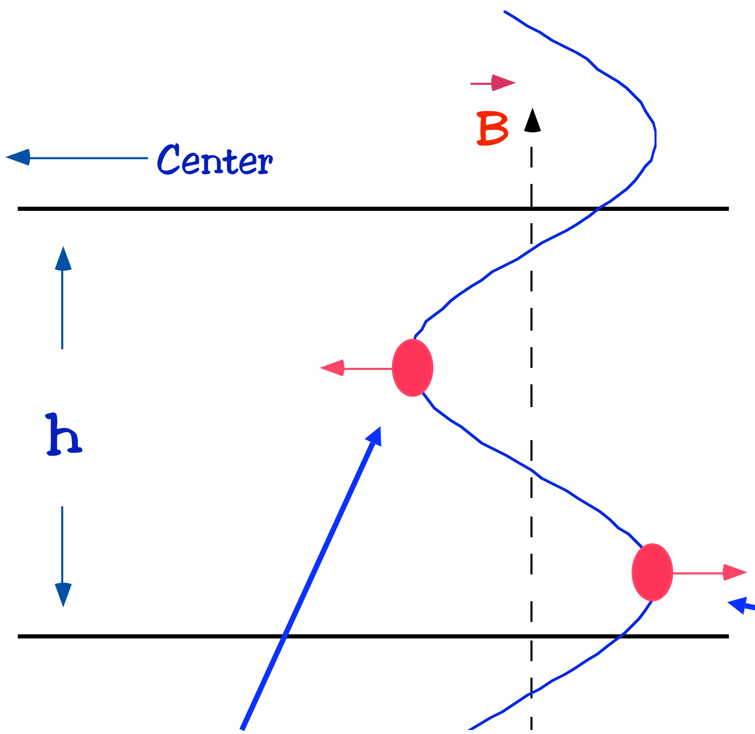
-> sinks further down

-> exponential growth !

-> extraction of magnetic flux from the Sun, galaxies... accretion disks ?

2°) Magneto-Rotational instability

in a differentially rotating disk



give fluid particles a radial kick

unmagnetized case: they retain their angular momentum

-> oscillations around their initial position at the epicyclic frequency κ (= Ω in keplerian disk)

magnetized case: the fluid particles stay tied to their field line

-> retain their rotation frequency

(exchange angular momentum by magnetic stresses)

this particle rotates too fast

-> tends to move further out

this particle rotates too slowly to fight gravity

-> tends to fall further in

-> the small initial motion is amplified

-> instability due ONLY to flux freezing

The Lorentz force

$$\vec{F} = \vec{j} \times \vec{B} = \frac{1}{4\pi}(\vec{\nabla} \times \vec{B}) \times \vec{B}$$

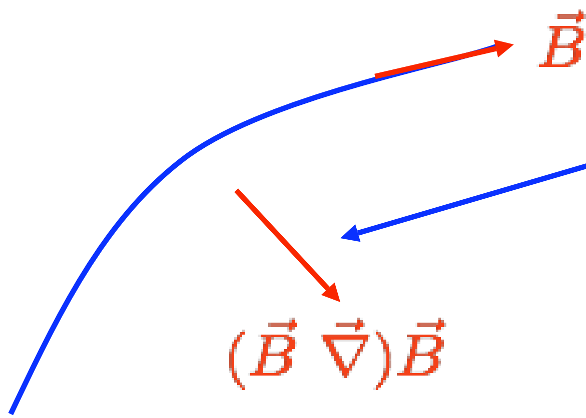
a small miracle of vector algebra,

... or rather: the subtle working

of the inner consistency of the equations of physics,

gives:
$$\vec{F} = -\vec{\nabla}\left(\frac{B^2}{8\pi}\right) + \frac{1}{4\pi}(\vec{B}\,\vec{\nabla})\vec{B}$$

magnetic pressure



magnetic tension (a tensor !)

a magnetic field line acts as an elastic rope

magnetic energy

2 forces -> 2 forms of energy :

- magnetic pressure $\dfrac{B^2}{8\pi}$

  compared to thermal energy density by $\beta = \dfrac{8\pi p}{B^2}$

NB: in astrophysics, since it is often impossible to measure B,
one commonly assumes equipartition: $\beta \sim 1$

this is not stupid!

e.g. interstellar medium : $p_{gas} \sim p_{mag} \sim p_{cosmic\ rays}$

because turbulence involving the 3 works toward equipartition

...but not universal ! e.g.solar corona

- magnetic tension (twisting of field lines)

-> new types of waves to propagate these new forms of energy

MHD waves

In an ordinary fluid, only 2 types of perturbations:

- sound waves (propagate pressure perturbations) -> $\omega^2 = k^2 c_s^2$
- vortices (vorticity, entropy) -> no propagation

In MHD, 3 waves:

- Alfvén wave: propagates magnetic torsion $\omega^2 = k_\parallel^2 \, V_A^2$

with $$v_A^2 = \frac{B^2}{4\pi\rho}$$

Note: $$\beta = \frac{8\pi p}{B^2} = \frac{2c_s^2}{v_A^2}$$

Equipartition -> $c_s \sim v_A$

only torsion, no compression in the wave

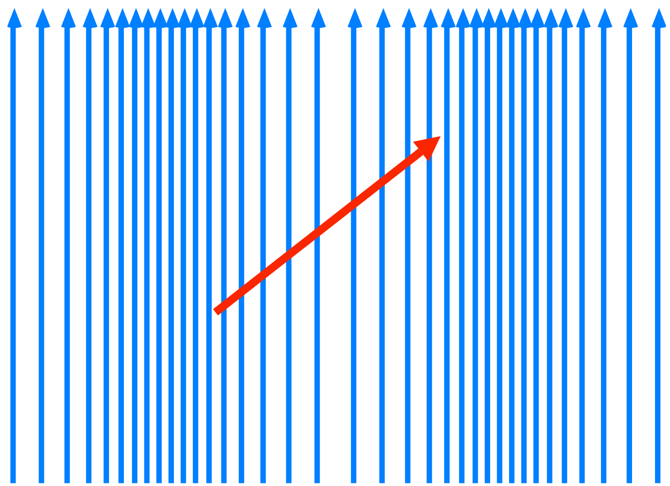
... and 2 magnetosonic waves

easier to discuss in one limit, e.g. $\beta \lesssim 1$

essentially:

- fast magnetosonic wave

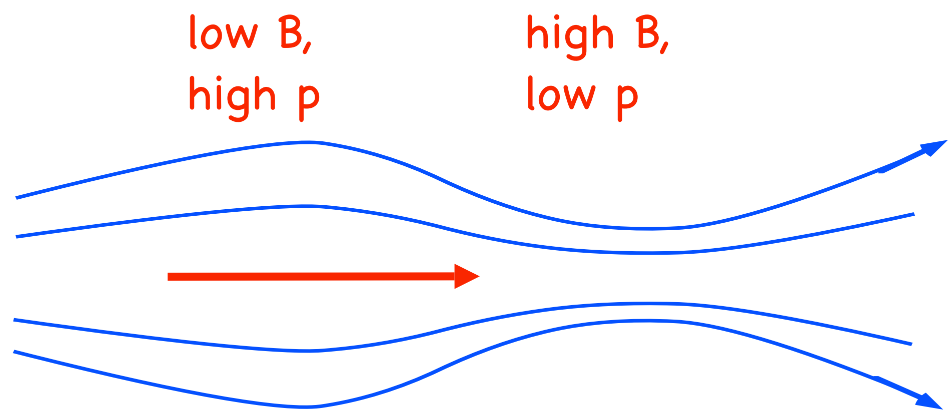
$$\omega^2 = k^2\,(v_A^2 + c_S^2)$$



propagates pressure (thermal + magnetic) in all directions

- slow magnetosonic wave

$$\omega^2 \approx k_\parallel^2\, c_S^2$$



equilibrates pressure(thermal + magnetic) along field lines -> maintains

$$p + \frac{B^2}{8\pi} = Constant$$

magnetic energy

- magnetic energy is stored as pressure or torsion of field lines
  <=> currents
- just as gradients of entropy, velocity, etc. this can cause
  instabilities = means to release the free energy
- thanks to the variety of waves, the magnetic field also provides
  new channels to release this free energy

(remember the MRI, due to gradient of Ω, i.e. gravitational energy
but exists only thanks to flux freezing)

- -> all the classical hydrodynamical instabilities (Rayleigh-Taylor, Kelvin-Helmholtz, etc.), modified by the magnetic field
- + new ones, due to the currents

MHD is all about where energy is,

where it can go,

how it can go there

and how it can be dissipated

to heat,

to radiation,

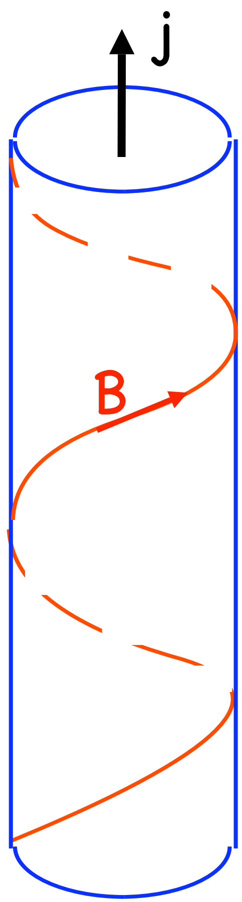
or to particle acceleration

e.g. kink instability

(applications to knots in jets)

a vertical field

+ a vertical current -> a helicoidal field

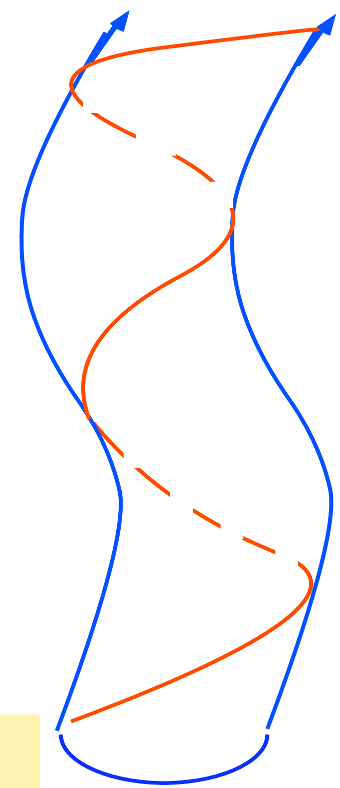


if the current becomes

too strong

(i.e. winds the field too much)

twisting of the whole configuration



as the elastic engine of a model airplane,
when it can't take more torsion energy

Reconnection

MHD often presses together regions of opposite magnetic field

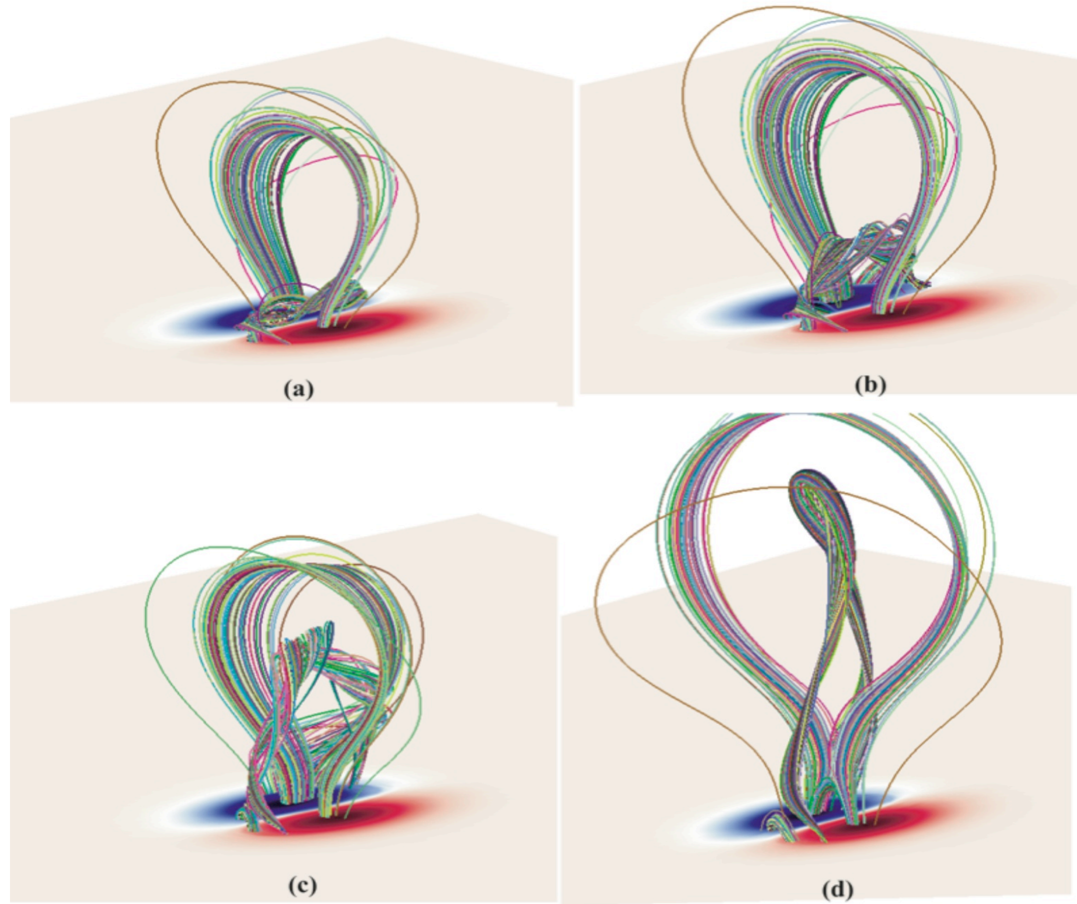
-> formation of current sheets

very strong j

-> ηj can act

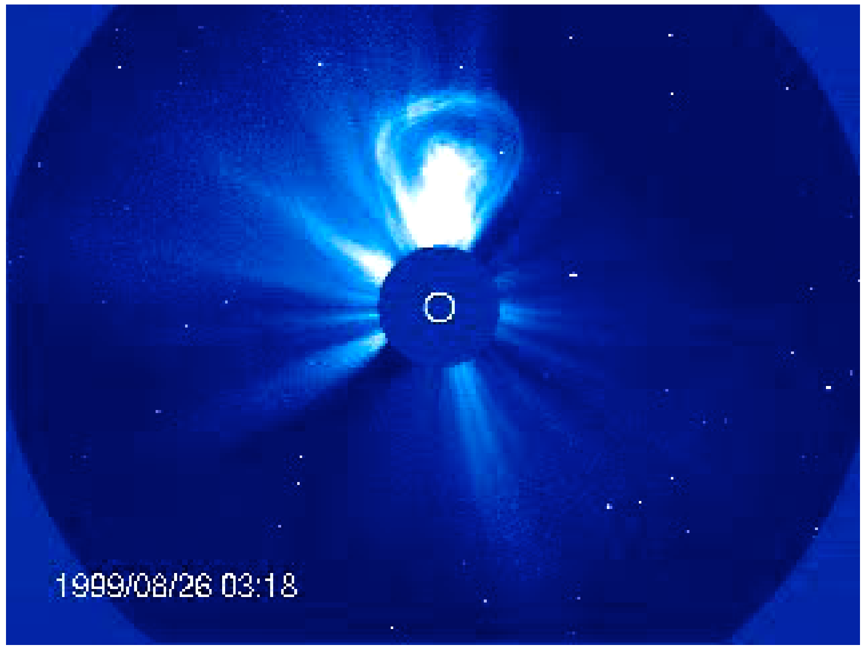
-> reconnection:

-> change of topology:



Amari et al.: simulation of a CME


16 February 1980: White Light
Source: High Altitude Observatory Archives
HAO A-009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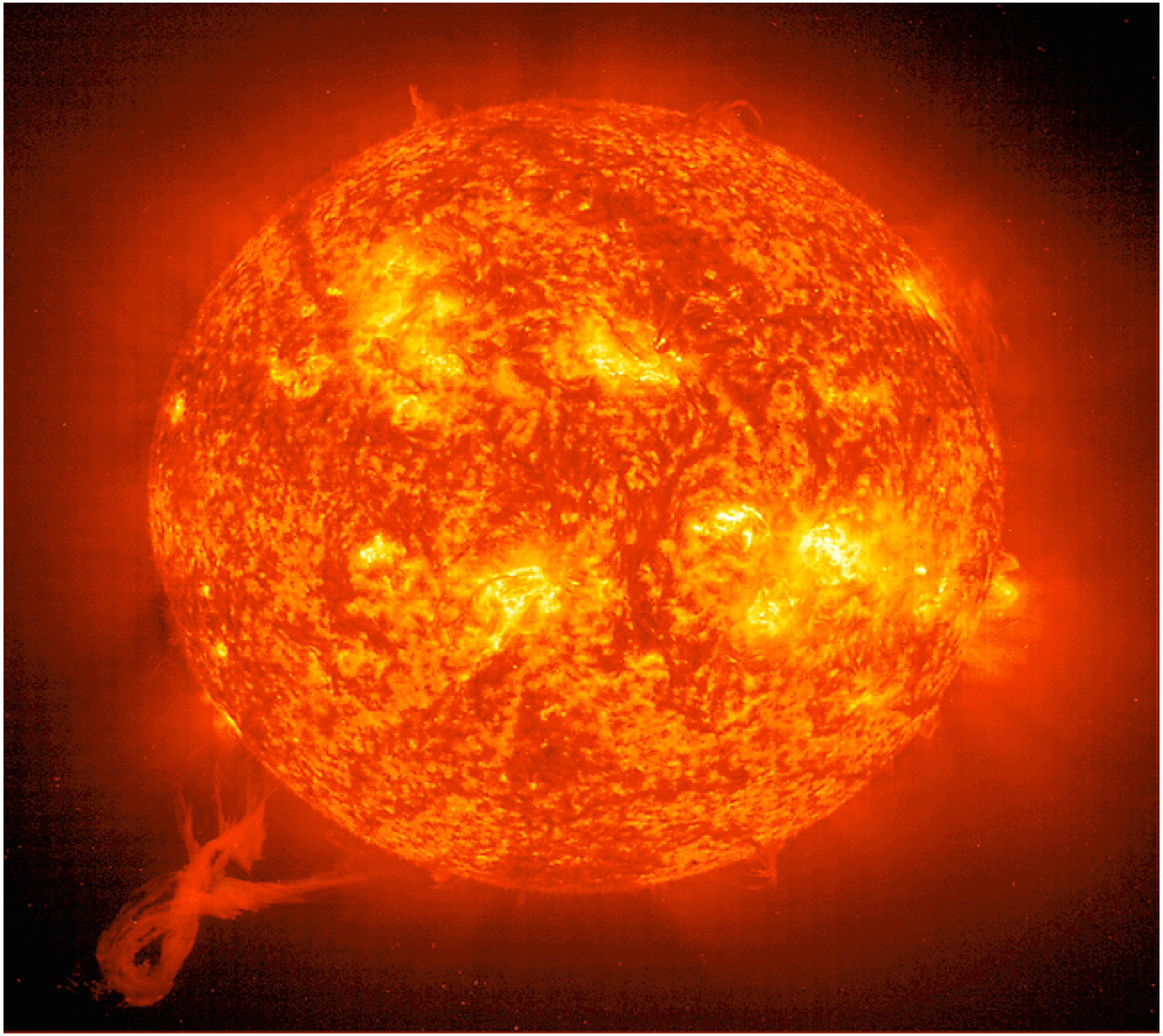
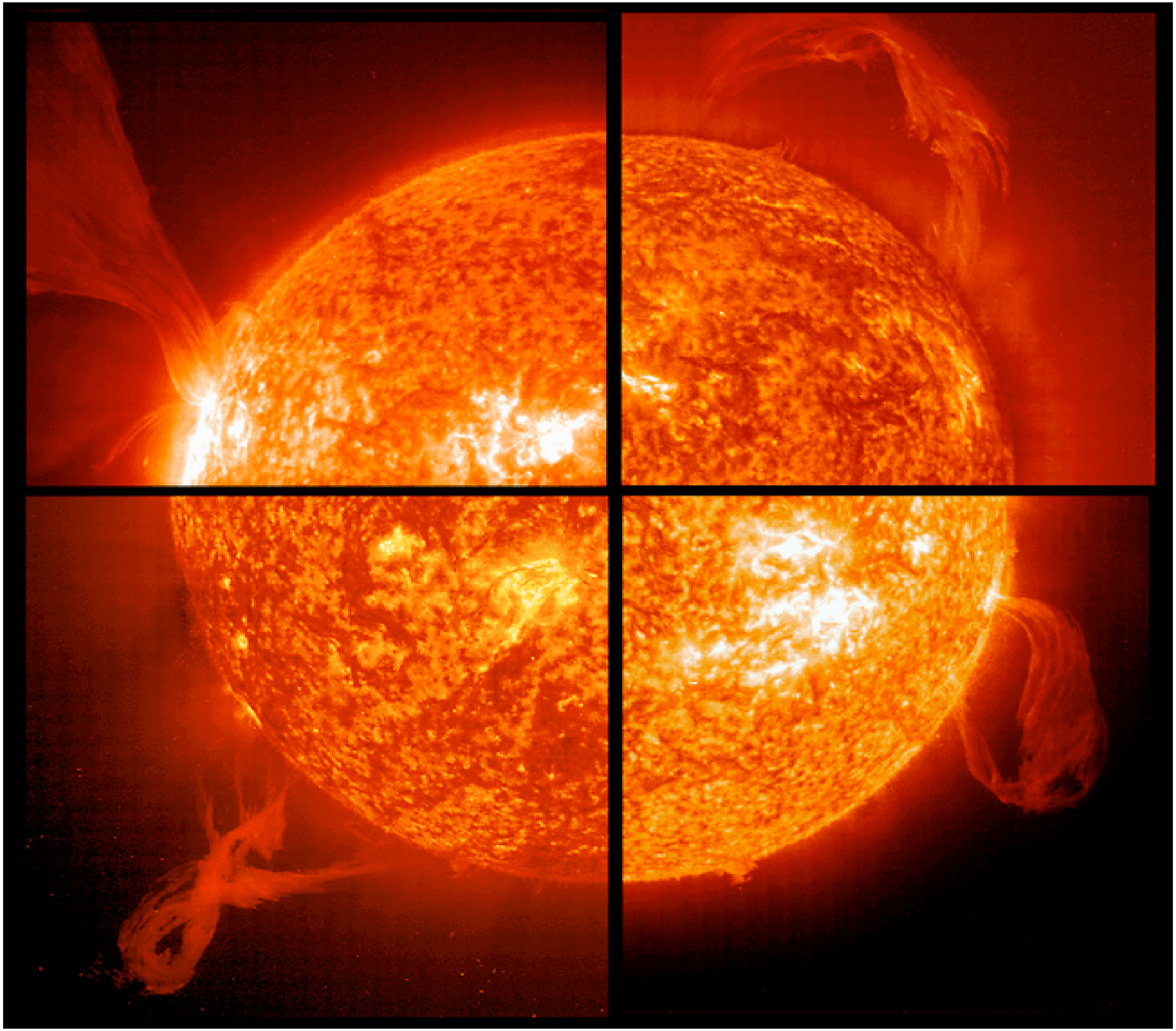
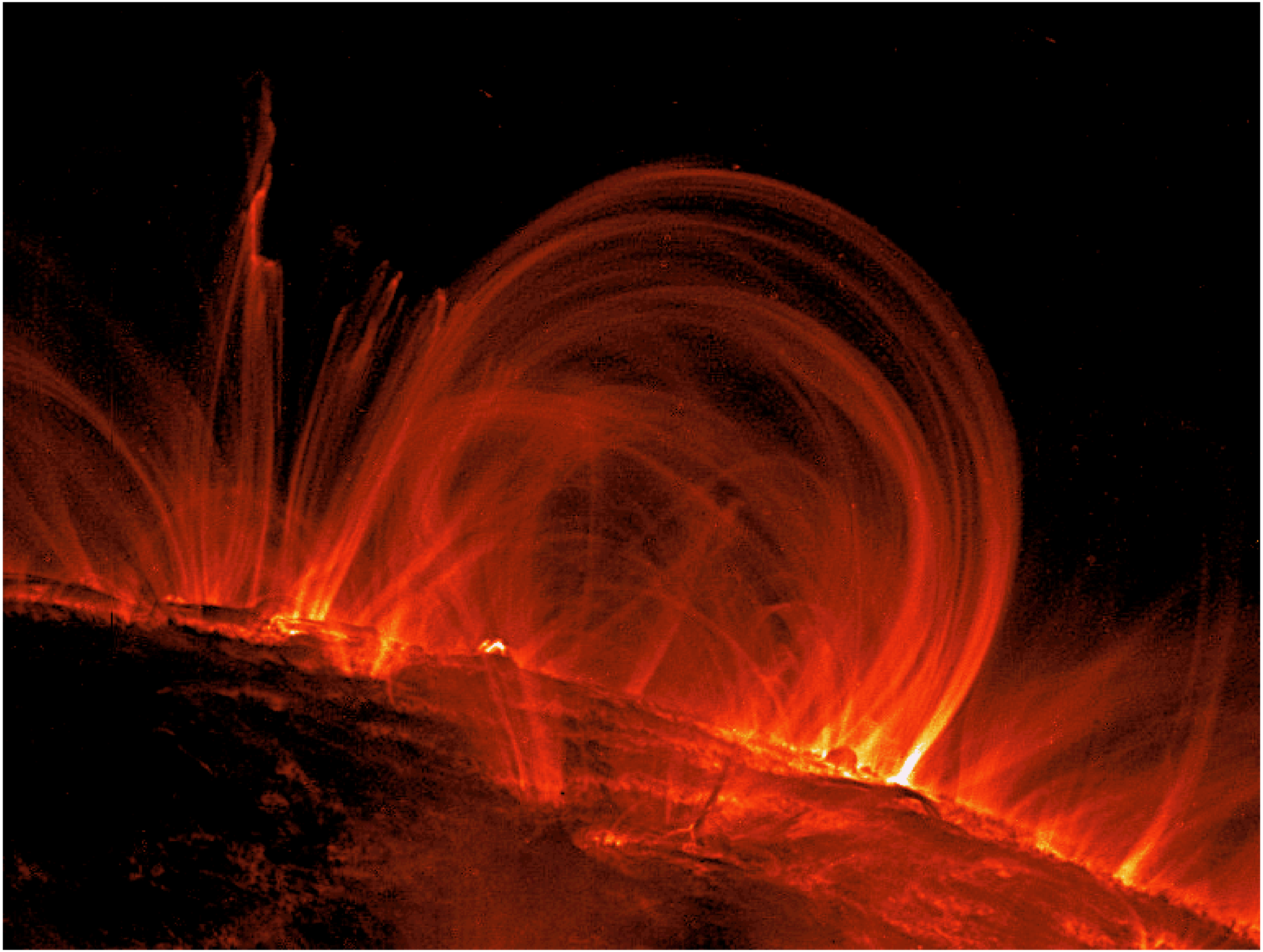
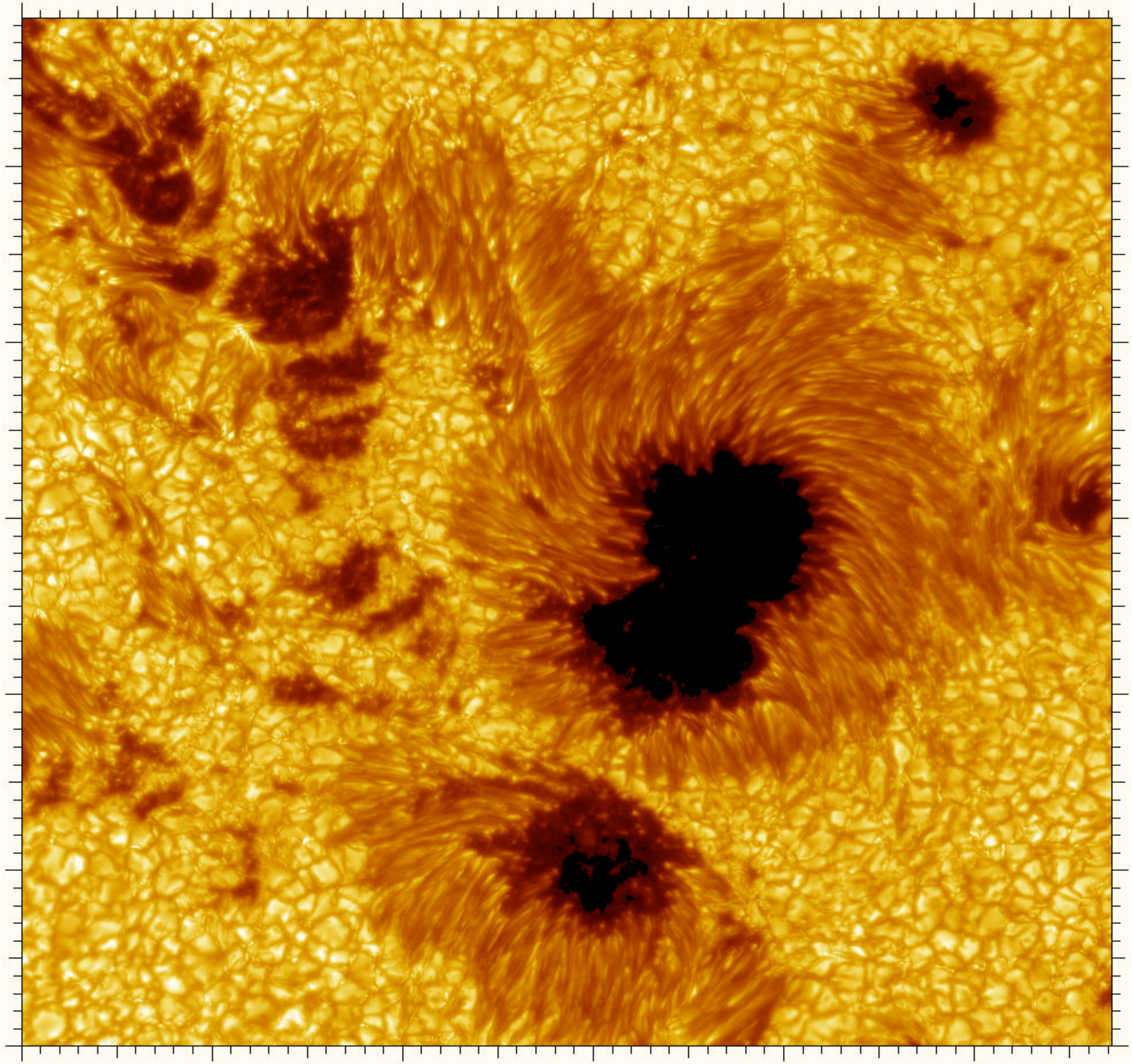
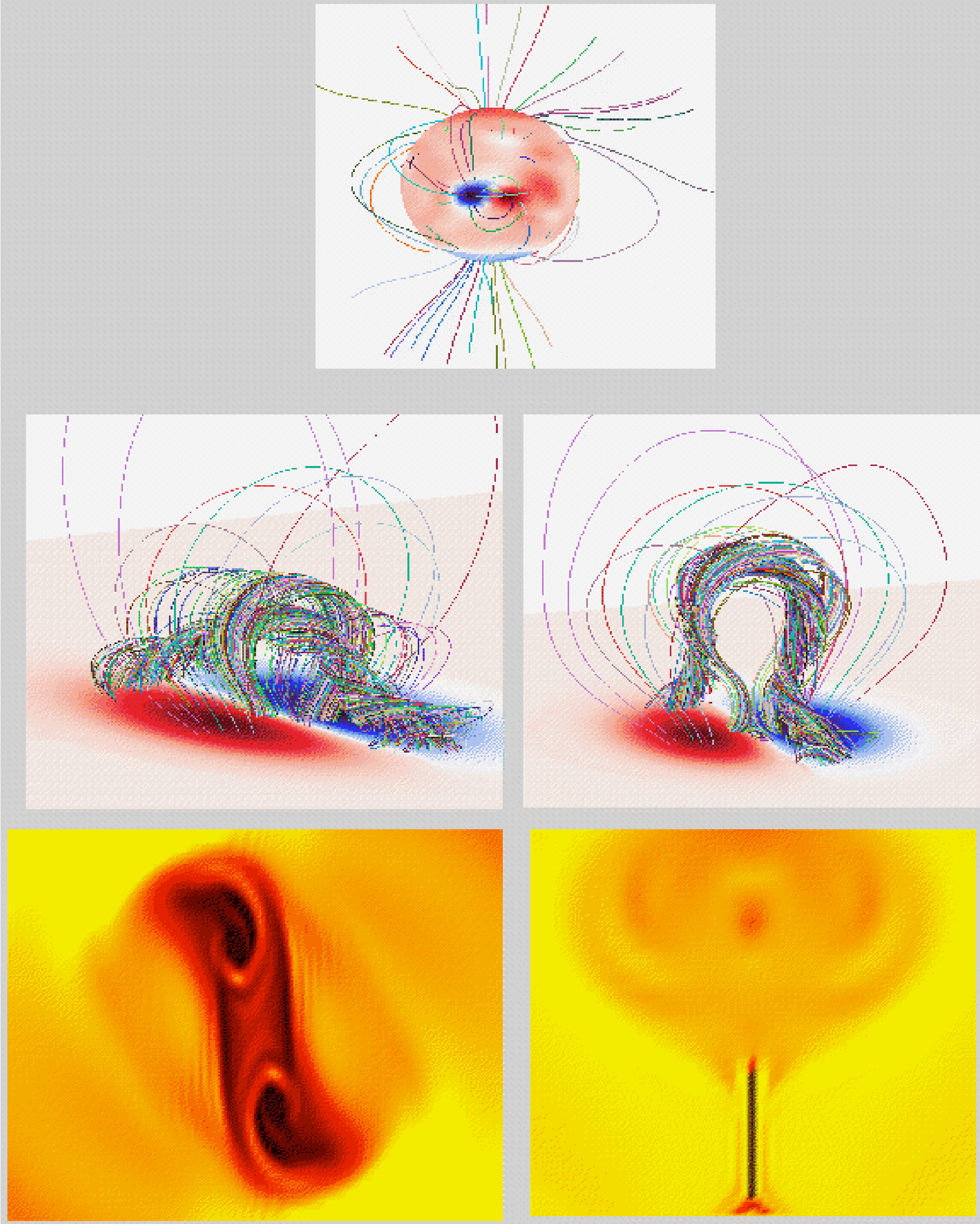


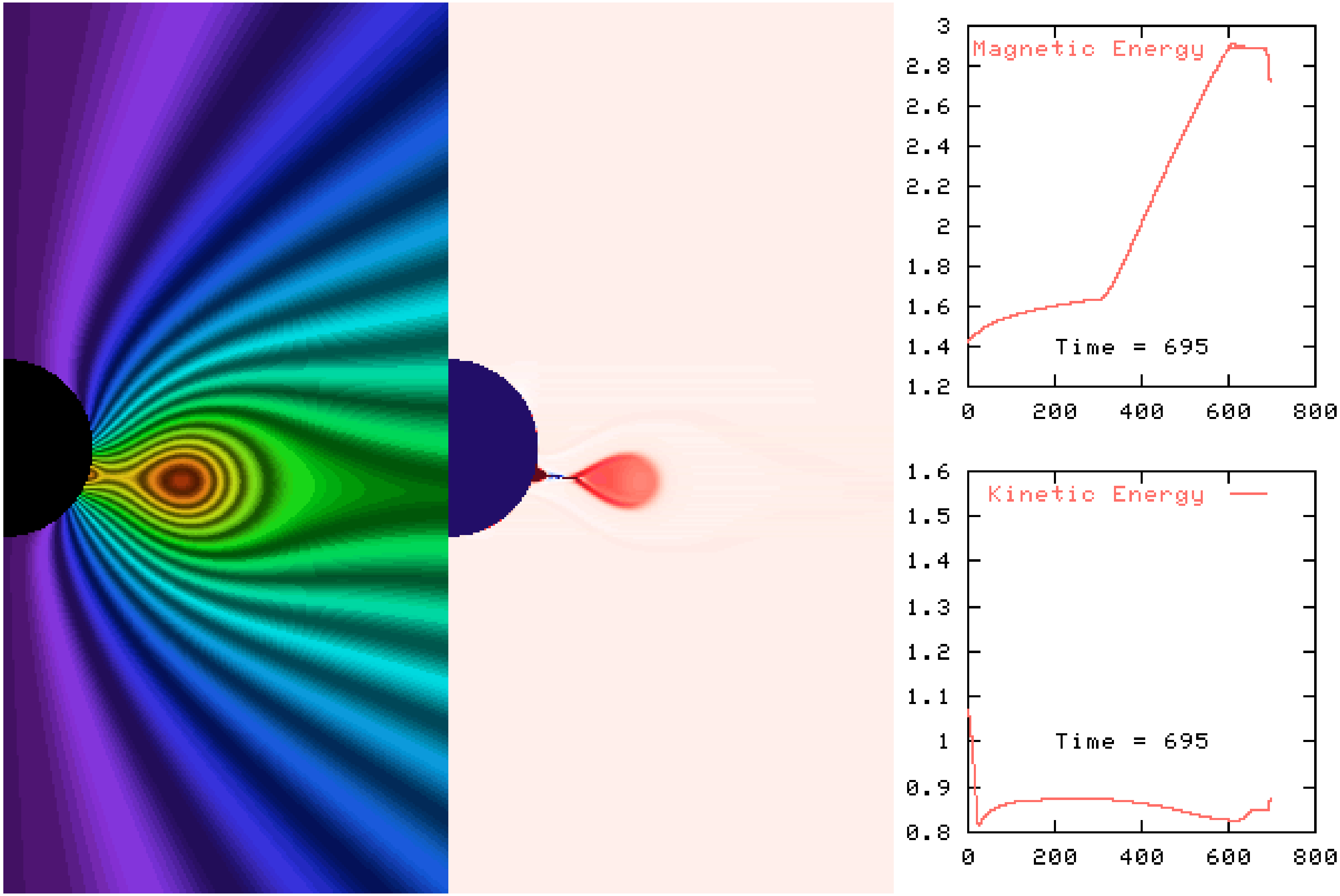








Amari, T., Luciani, J.F, Mikic, Z. and Linker J.



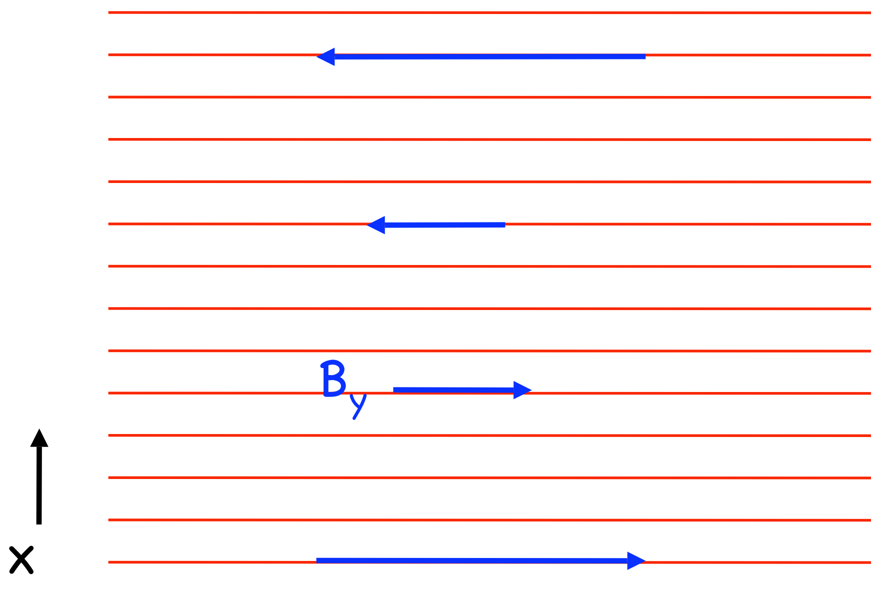
magnetic energy and flux from inside the sun emerge in the corona (Parker)

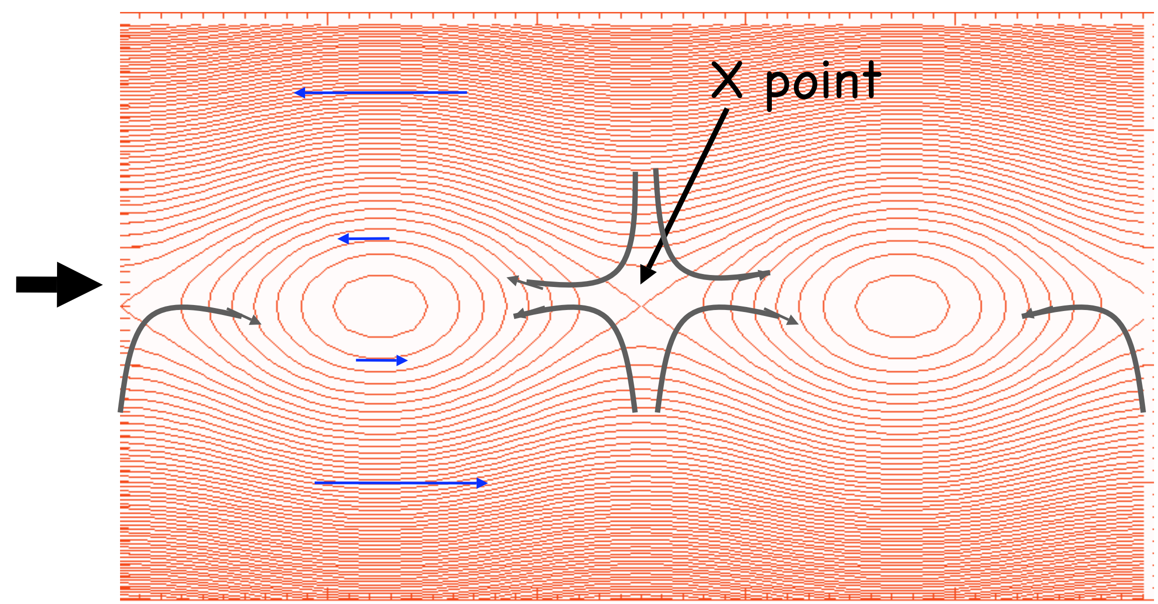
energy stored in magnetic loops is suddenly released as kinetic energy (+ heat)

reconnection: a simple case

(tearing instability)

- ideal MHD often brings together regions of opposite B
- -> current sheets -> resistivity can act





$$\frac{dB_y}{dx} = \frac{4\pi}{c} j_z$$

(plus a constant Bz)

an ideal MHD instability compresses the field lines, resistivity allows the formation of magnetic islands -> destruction of magnetic flux

Reconnection (continued)

(need a few equations...)

- use a simple description: $\vec{B} = \vec{e}_z \times \vec{\nabla}\psi(x,y,z)$ → $B_x = -\dfrac{\partial \psi}{\partial y}$, $B_y = +\dfrac{\partial \psi}{\partial x}$

- equilibrium: $\psi(x,y,z) = B_0 \dfrac{x^2}{2}$ → $B_y = B_0\, x$

- note: ψ = constant along field lines

  = magnetic flux (per unit length) below a field line

- from Ohm's law (induction equation) $\vec{E} + \dfrac{1}{c}\vec{V} \times \vec{B} = \eta\, \vec{j}$

one gets $\dfrac{\partial \psi}{\partial t} + \vec{V}.\vec{\nabla}\psi = \eta \Delta \psi$ => $\dfrac{d\psi}{dt} = \eta \Delta \psi$

local variation + variation along the gas trajectory

total variation with the gas flow

reconnection...

- ideal MHD: $\frac{d\psi}{dt} = 0$ frozen magnetic flux
- resistive MHD: $\frac{d\psi}{dt} = \eta \Delta \psi$ diffusion of magnetic flux

η = resistivity = magnetic diffusivity, extremely small in most plasmas

Text e

$\Rightarrow$ MHD has to produce extremely thin regions with very strong gradients

$\Rightarrow$ ($\frac{\partial}{\partial x} \sim \eta^{1/2}$) for resistivity to act => often very slow

and extremely localized

One often invokes "turbulent resistivity"

just as turbulent viscosity

this is just as convenient

BUT EVEN MORE MISLEADING

(NEVER true in laboratory plasmas)

non-ideal MHD

- ideal MHD: $\vec{E} + \frac{1}{c}\vec{V} \times \vec{B} = 0$ => $E_{\|} = 0$
- resistive : $\vec{E} + \frac{1}{c}\vec{V} \times \vec{B} = \eta\,\vec{j}$ => $E_{\|} \neq 0$

=> possibility to accelerate particles along field lines (if not too collisional)

=> e.g. solar flares

=> already non-MHD effects ! (some) energy can go to particle acceleration (ordered motion) rather than heat

=> kinetic effects may affect the dynamics in the reconnection layer in a manner that differs strongly from resistivity

$\Rightarrow$ believing in turbulent viscosity is already bad manners
if you believe too much in turbulent resistivity,
don't pretend you haven't been warned !

lesson: if you want to be able to assess critically and constructively MHD results, the SECOND BEST WAY is to count a plasma theorist among your very best friends (the very best is to hire one)

auroras are the result of non-ideal MHD processes in the tail of the magnetosphere,

accelerating particles along magnetic field lines.

the theory (still incomplete) is probably even more beautifull than the real thing

bad news...

"he" called and said he wasn't interested

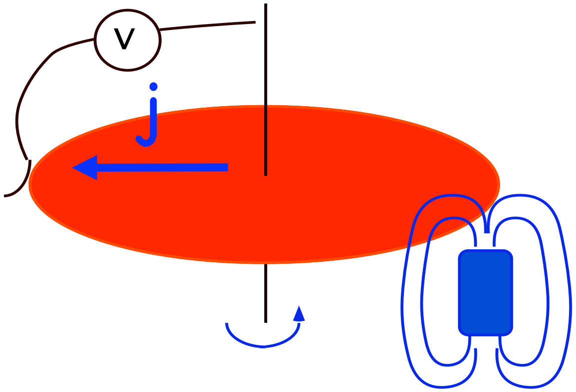
in the 25h MHD class

so I will have to dump that course on you before friday

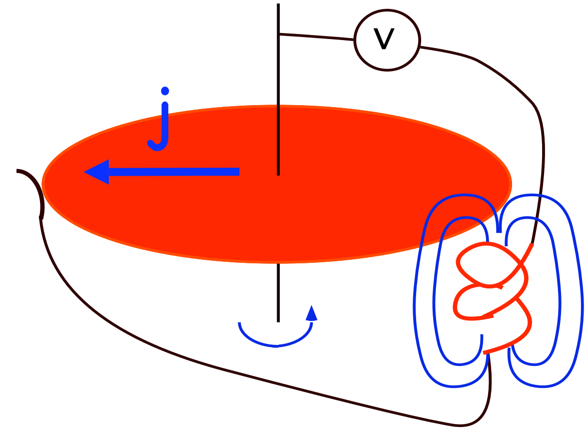
unless we try with this guy ?

new concept: the totally masked coded mask

turbulent dynamo



simple dynamo



self-excited dynamo

generates its own B

from any small ("seed")

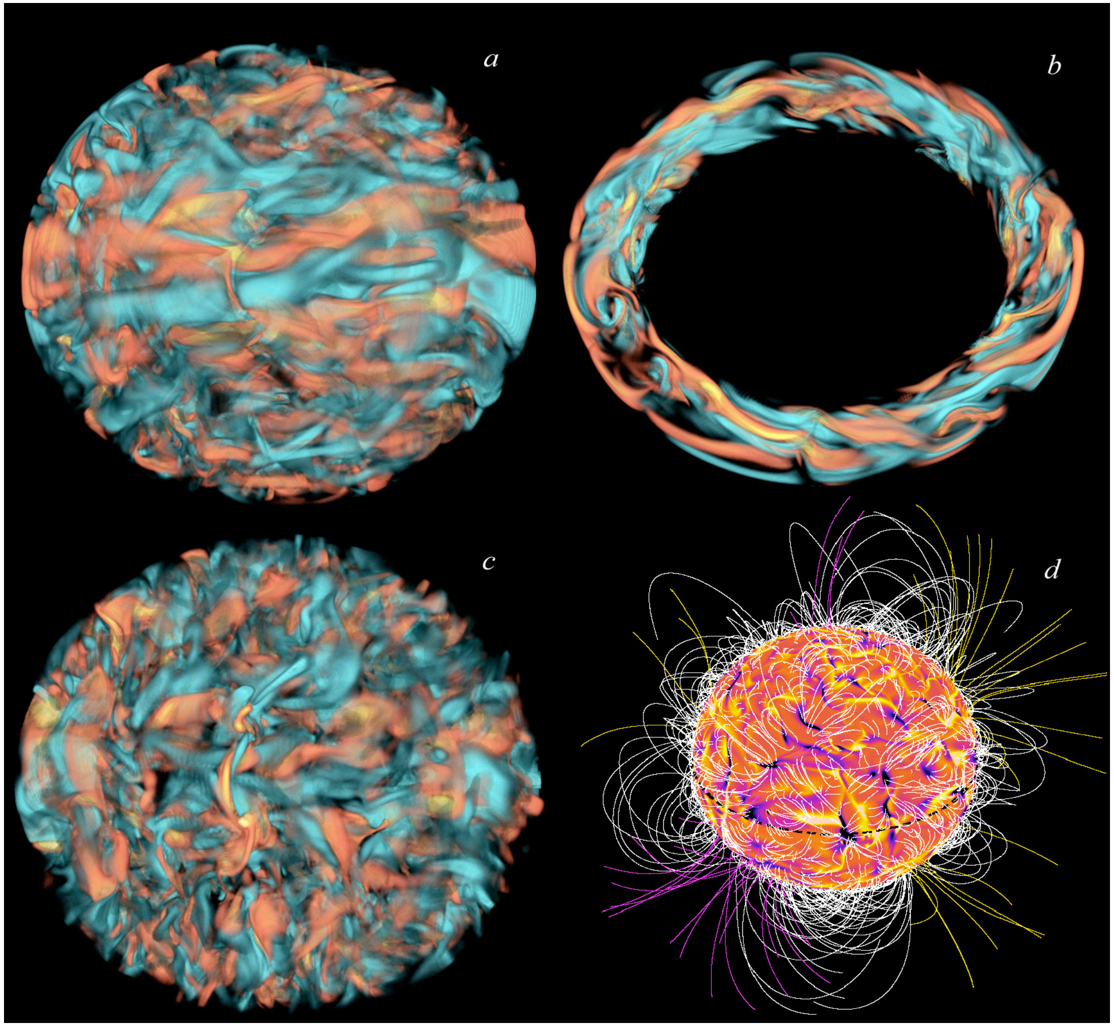
magnetic field

turbulent dynamo (Earth, Sun, galaxies (?), accretion disks (?)):

If the turbulent velocity field has the right property ("helicity")

it can generate a large-scale magnetic field

-> conversion of kinetic to magnetic energy



Brun et al.:
simulation of the
solar dynamo

a word about MHD simulations

- problems due to the discretization:

$$\frac{df}{dx} \rightarrow \frac{f(x+dx)-f(x)}{dx} = \frac{df}{dx} + (....)f$$

- maintain $\vec{\nabla}.\vec{B} = 0$ (some people don't! -> accretion on monopoles ?)

-> constrained transport (ZEUS) still used in more modern codes

but adds complexity; other methods...

- stability issues (since various waves, sometimes at very different speeds)

- resolving very small scales, boundary conditions, initial equilibrium...

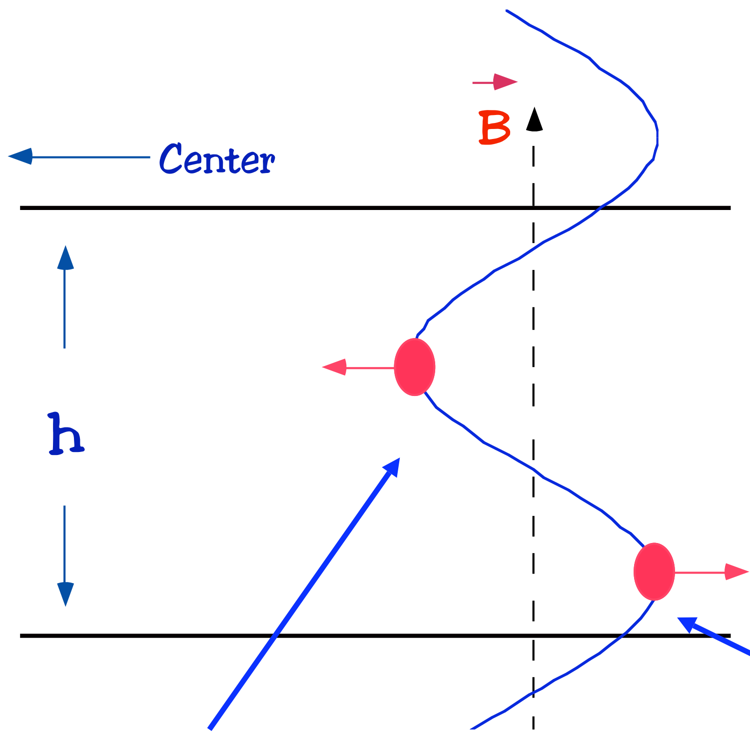
- numerical "resistivity" and "viscosity" (more diffusive -> better movies!)

- more modern codes under development (VAC, Athena, Astro-Bear, Ramses-MHD...): Godounov algorithm (-> 2nd order precision, solves shocks and discontinuities...). BUT exact Godounov scheme for hydro, only approximate ones in MHD (NO universal one)
- Adaptive Mesh Refinement

dynamical disk instabilities

- need to explain / understand the turbulence causing accretion
- generally turbulence is due to external forced motion (e.g. an object moving in a fluid)
- or to instabilities: extract free energy from the fluid (due to gradients of density, entropy, to currents...) and transform it into kinetic energy (turbulent motion) -> thermalized as heat -> decreases the gradient
- e.g. convection transports heat better than diffusion
- example of a disk instability : galactic spirals, due to self-gravity
- but self-gravity is too weak in accretion disks
- => stuck for a long time
- 1991: Balbus and Hawley => Magneto-Rotational Instability
- (1990: Tagger et al.: magnetic spiral instability, but too weak)
- 1999: Tagger& Pellat, Accretion-Ejection Instability, (usefull) complement to MRI
- + much additional work...

Magneto-Rotational Instability



this particle rotates too slowly to fight gravity

-> tends to fall further in

this particle rotates too fast

-> tends to move further out

this perturbed motion releases

gravitational energy $\propto \rho \frac{\partial}{\partial r}(\Omega^2)$

but also costs magnetic energy to bend the field lines

$$\propto k_z^2 B^2 \propto \rho\, k_z^2 v_A^2$$

-> instability criterion:

$$r \frac{\partial}{\partial r}(\Omega^2) + k_z^2 v_A^2 < 0$$

on the other hand $(k_z h)^2$ must be > 1 to fit in the disk, and (see King's lecture) $h \sim c_S/\Omega$. Using the expression for v_A^2 the criterion boils down to:

$$\beta = \frac{8\pi p}{B^2} > 1$$

-> any weakly magnetized disk !

MRI (continued)

main successes:

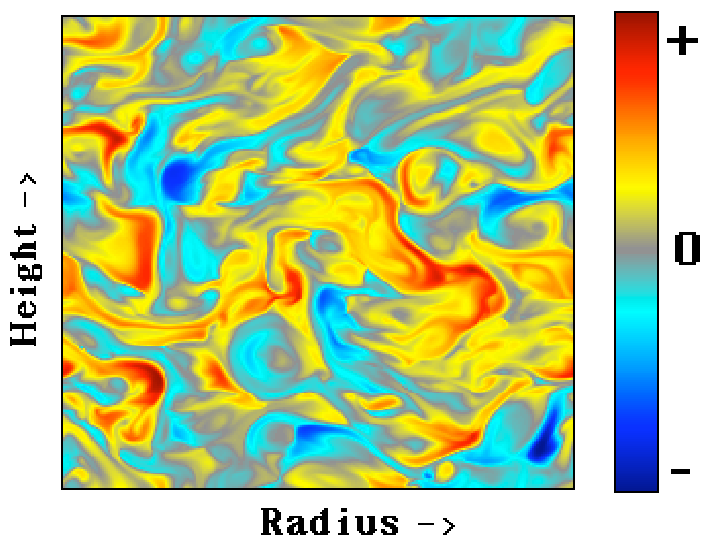
- explains turbulence in any magnetic geometry, any magnetic field < equipartition
- causes accretion at reasonable rate (NOT α viscosity...)
- generates by itself strong magnetic field

- however:
  - jets ?
  - no QPO, even in GR - MHD
  - constraint on numerical simulations:

    no net vertical magnetic flux in the disk

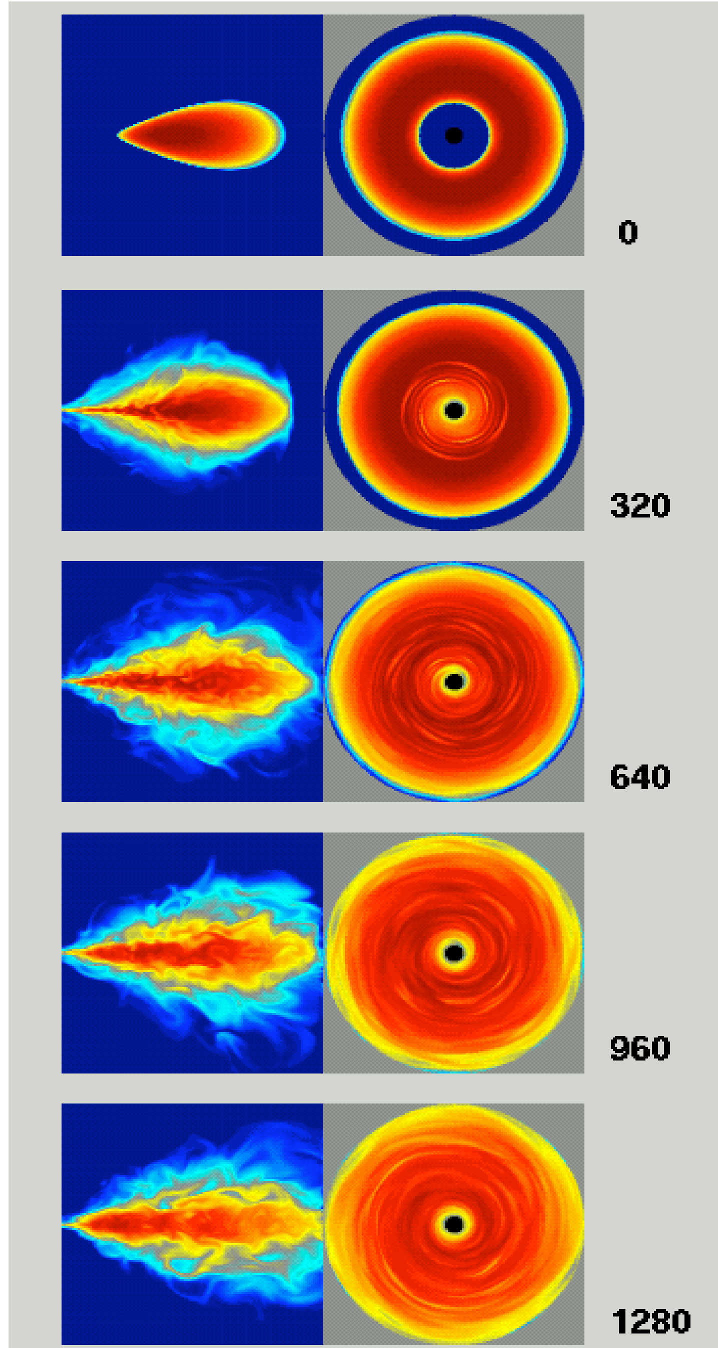
numerical simulations of the MRI

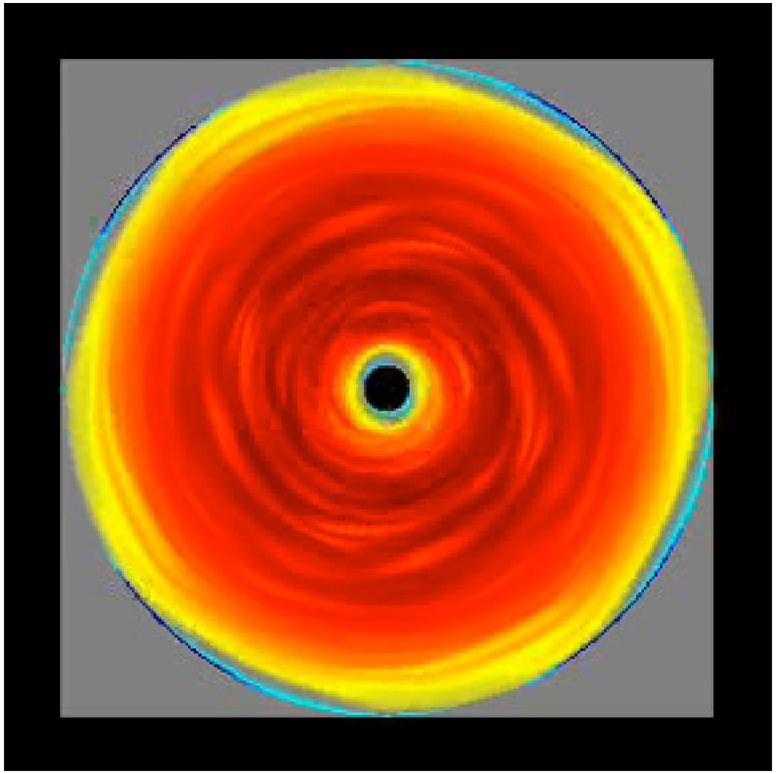
(Hawley and co-workers)

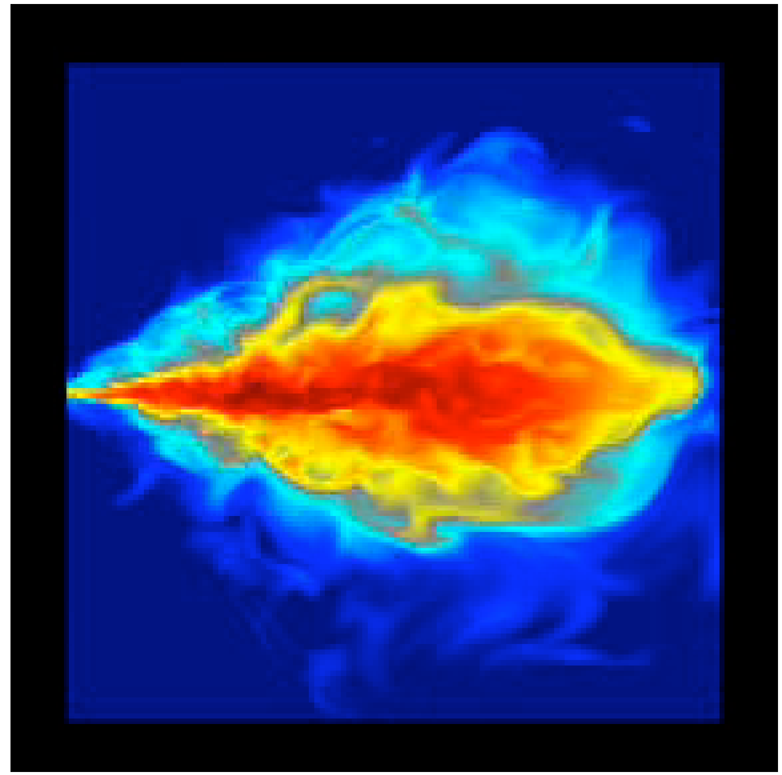


local

Global 3D

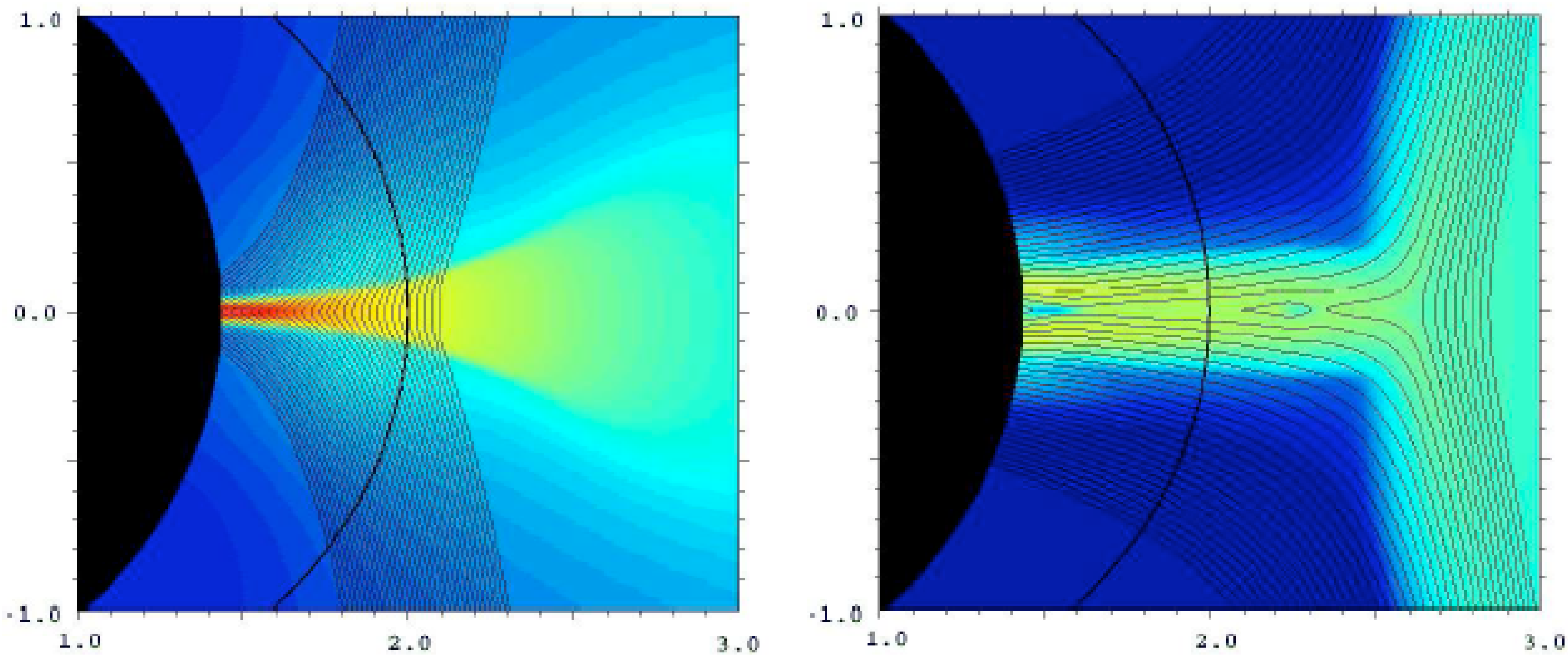






Hawley et al.

a taste of GR-MHD



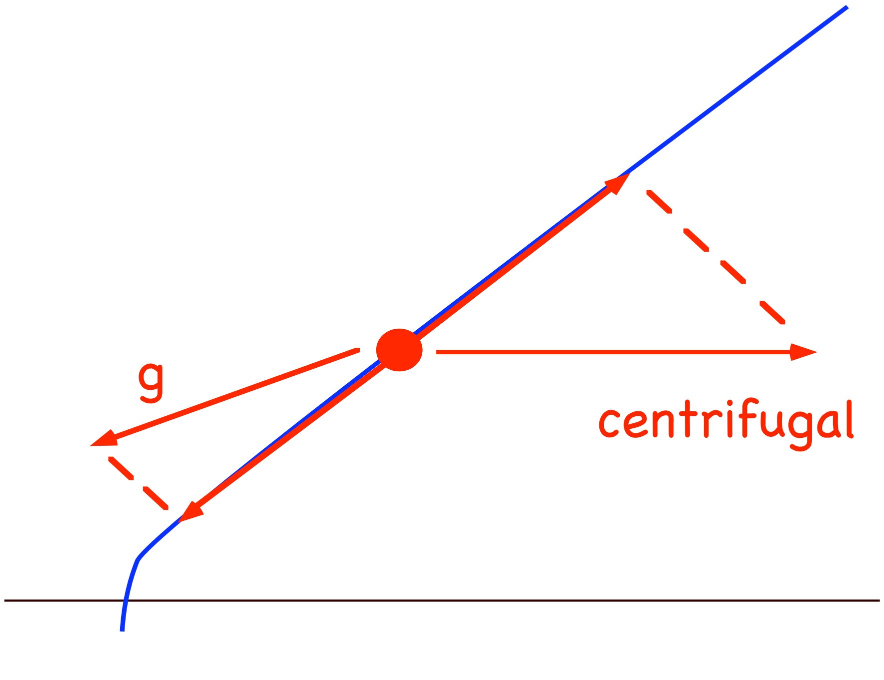
Kommissarov: axisymmetric simulation of the Blandford-Znajek and Penrose processes. B allows to extract energy from a spinning Black Hole

... but doesn't seem efficient enough for jets

MHD jet models

Blandford&Payne, Lovelace, Pelletier+co-workers...

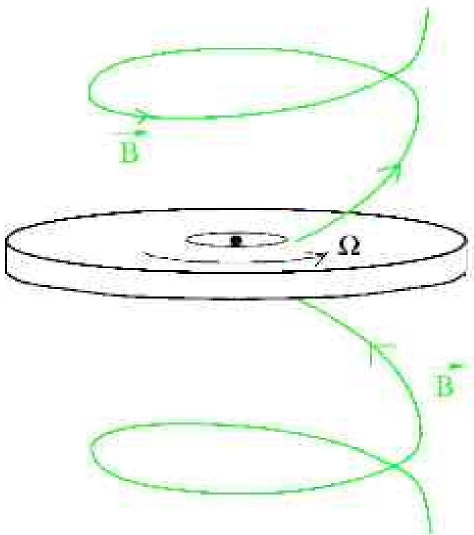
If the disk is threaded by a vertical field



- because fluid particles have to move along field lines, and to rotate at their angular velocity
- project the forces along field lines =>
- centrifugal force wins if the field line is inclined more than 30° to the vertical (beads on a wire..., Henriksen)
- fluid particles are accelerated, while the whole field line still rotates at the same Ω

-> allows to extract a lot of angular momentum with ejection of only a little gas, and self-collimation

self-collimation of MHD jets



- When the azimuthal velocity $r\,\Omega$
  becomes larger than v_A

=> kinetic energy > magnetic energy

=> the gas is able to bend the field lines

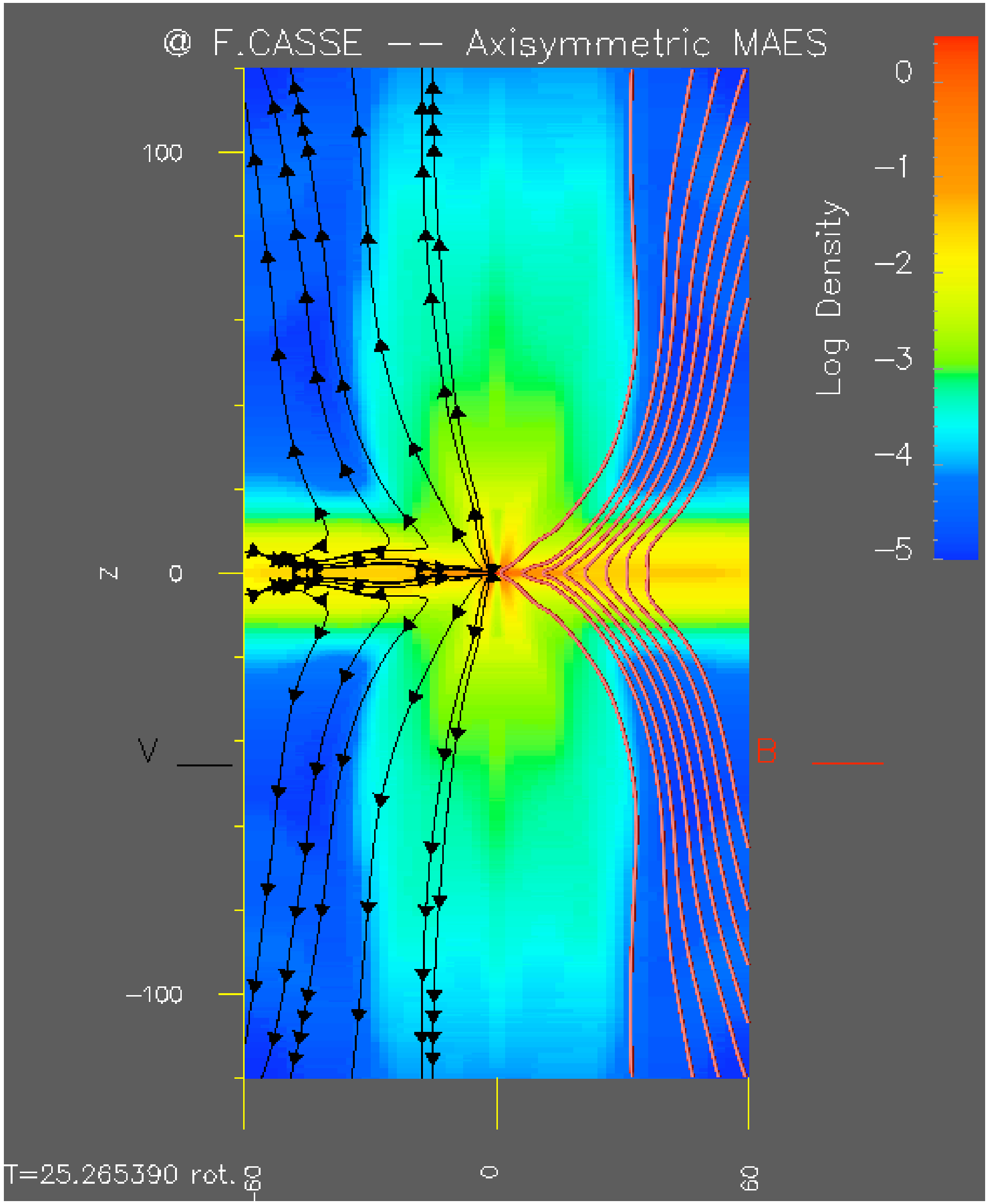
so that moving up limits its angular velocity

(as on a corkscrew)

then $B_r \longrightarrow B_\vartheta \longrightarrow j_z \longrightarrow \vec{j}_z \times \vec{B}_\vartheta$ (hoop stress)

-> SELF-COLLIMATION !

to explain the long, thin jets



magnetized accretion-ejection structures

connecting disk to jet

(Pelletier, Ferreira + coworkers)

(connection to the disk - => requires $\beta \sim 1$)

F. Casse

(axisymmetric simulation

-> no turbulence

-> using "anomalous resistivity" to allow continuous description from the disk to the jet, following analytical results

Quasi-Periodic Oscillations and normal modes

- some QPO models -> only predict frequencies

  -> have to assume orbiting "blobs"
- but blobs would be sheared away by differential rotation

  in ~ 1 rotation time
- whereas QPOs are (quasi-)coherent
- an alternative: normal modes = standing wave patterns

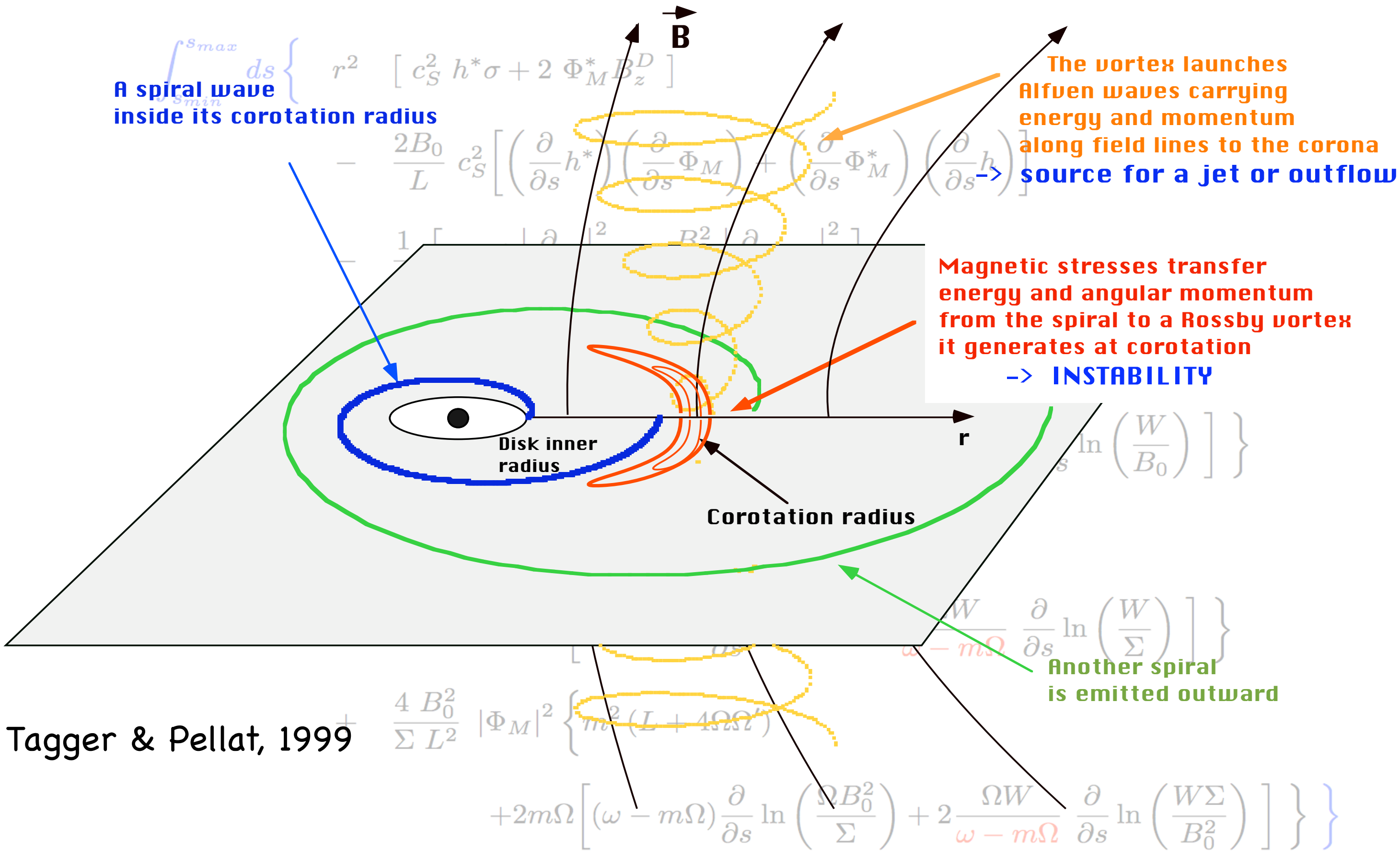
  (as in any cavity, i.e. waveguide, microwave oven, the Sun, a bell...)

  -> seismology
- but need for external excitation (hammer on the bell)
- whereas the LF-QPO can reach 40% RMS (BIG hammer needed!)
- best possibility: UNSTABLE NORMAL MODES

  as the (barred) spiral in Galaxies
- what is the best means to make them unstable ?

the magnetic field !

Accretion-Ejection Instability



Tagger & Pellat, 1999

A normal mode

(as in seismology, waveguides, microwave ovens,

the bridge on Saturday...) i.e.

a single frequency standing wave pattern

which is unstable

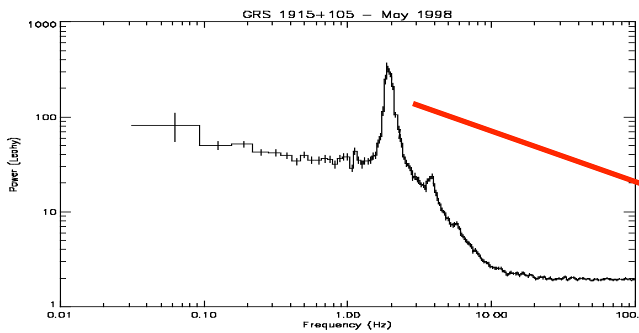
i.e. self-excited,

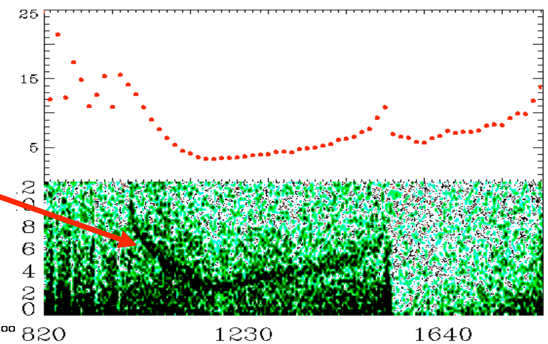
no need for external excitation

- in the conditions required by jet models (vertical field, $\beta \sim 1$), a spiral instability near the inner edge of the disk
- can redirect a significant fraction of the accretion energy upward as Alfvén wave (whence its name) though not a jet yet !

AEI (continued)

- with J. Rodriguez, P. Varnière : possible explanation for the low-frequency QPO

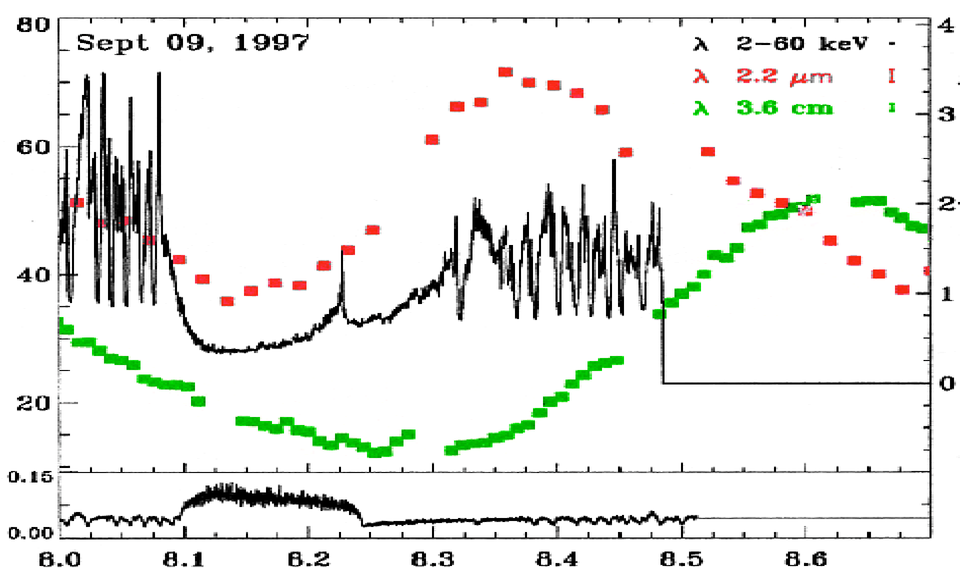




-> "Magnetic Floods" scenario for the cycles of GRS 1915+105:

-> controled by the advection and cycling of vertical magnetic flux

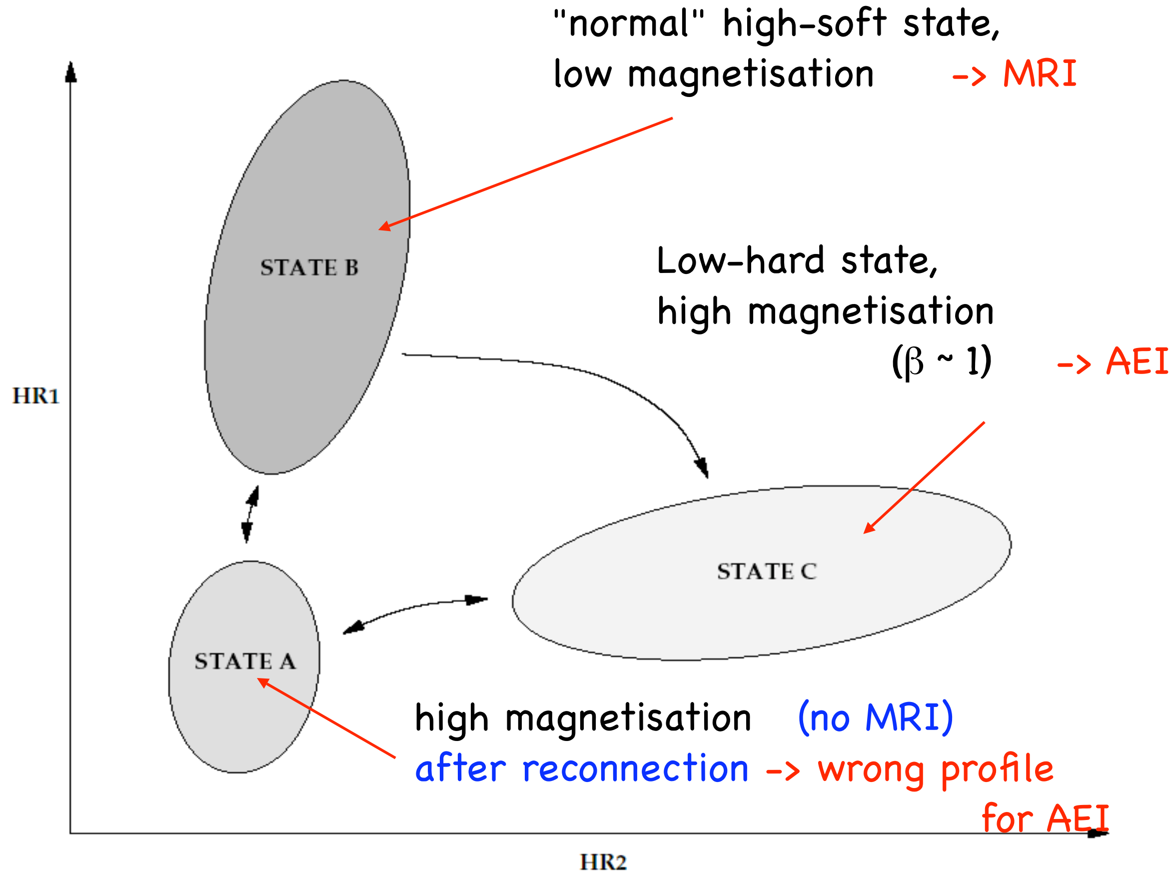
Markwardt



Chaty (PhD thesis), Mirabel et al.

AEI (continued)

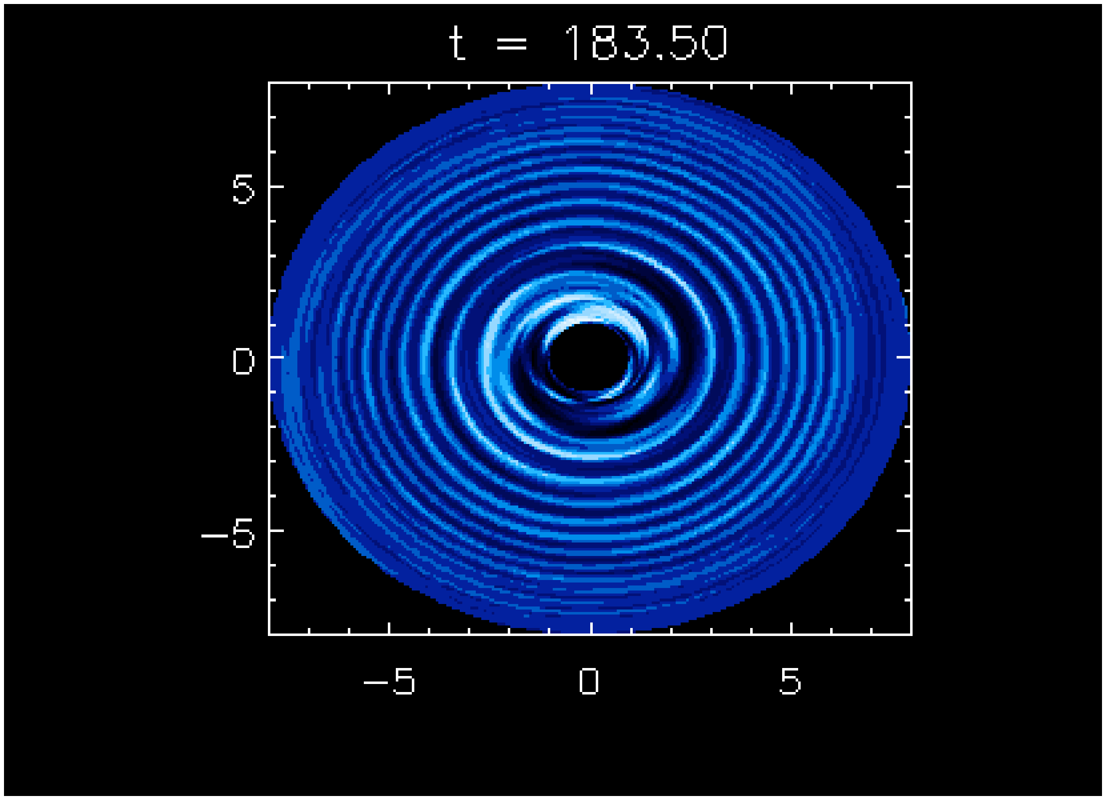
added bonus: the "forbidden" transition of Belloni et al.:



Magnetic floods scenario : cycles controlled by the cycling of vertical magnetic flux in the disk and the central hole

2D numerical simulation of the AEI

(2D well adapted because thin disk, n_z=0 mode)



Caunt & Tagger

A spiral wave appears with initially 3 arms

(depends on initial conditions...)

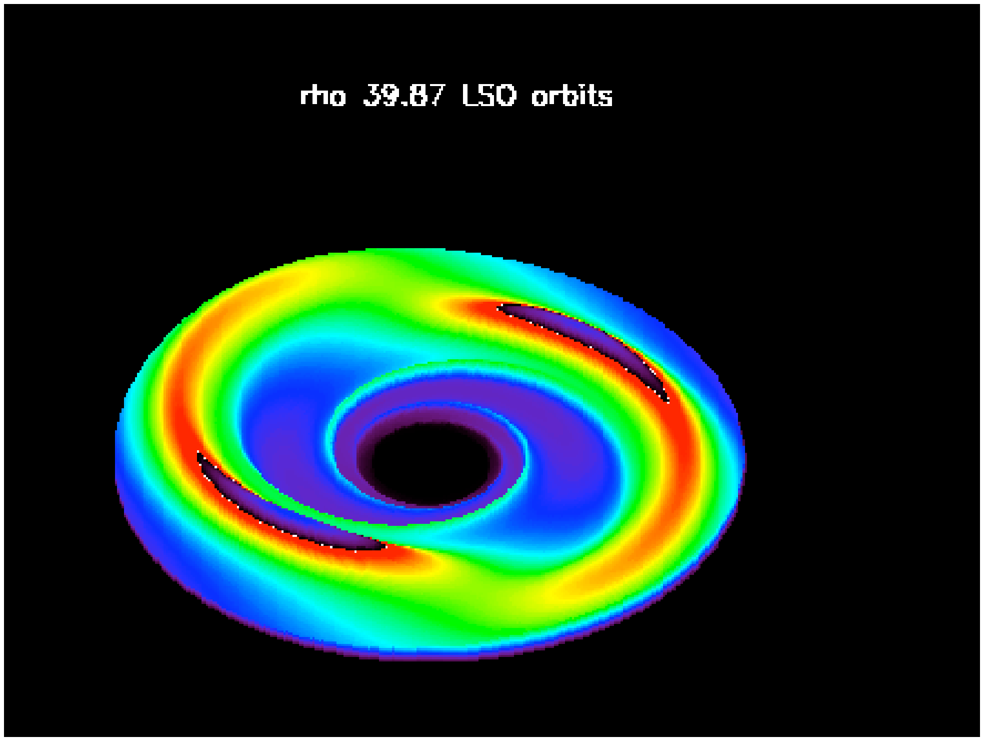
then 2 and finally 1 arm

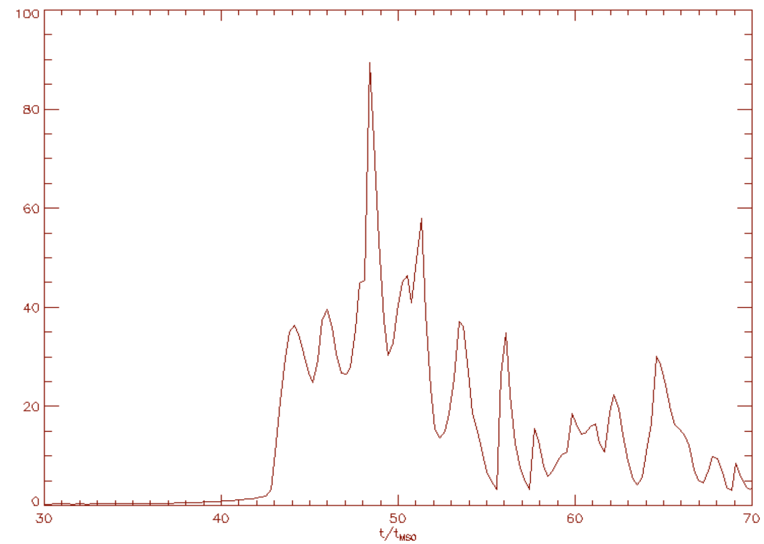
As gas and magnetic flux accumulate in the inner region

The flares at the Galactic Center

A different but closely related instability (RWI: Lovelace et al.), in its MHD form:

If a gas blob joins the disk and circularizes at a few tens $r_{Schwartzschild}$





accretion rate

at the inner edge, not unlike the IR and X-ray lightcurves

(see Eckart's talk)

Tagger & Melia, 2006

next stage

- HF-QPO of BH binaries
- coming soon

I was supposed to write the paper here
but the other talks are too
interesting !